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Self-induced thermocapillary convection in film boiling heat transfer from a vertical surface in saturated conditions and viscous regimen

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ABSTRACT

Self-induced thermocapillary convection and its significance with regard to film boiling heat transfer from a vertical saturated surface is discussed. Utilizing a simplified geometrical model, an analytical expression (multiplier factor) for the vapor film thickness and heat transfer coefficient corrected by thermocapillary phenomena was derived. The above equation is a new theoretical viewpoint for the enhancement in the heat transfer coefficient observed in the experimental data in the viscous regime and agree qualitatively with available experimental measurements made on R113 coolant.

1. Introduction

Whereas a lot of research has been done on nucleate boiling of various fluids (Fujita, 1992), and film boiling (Gian et al., 2009; Arias and Reventos, in press-a; Sarma et al., 1994), and special situations as electrical and magnetohydrodynamics aspects (Verplaetsen and Berghmans, 1999; Arias, in press-b). As regards the thermocapillary convection under imposed temperature gradients, in nucleate boiling we have the works from Zell (1991), Vogel (1994), and the recent works of Bennacer et al. (2004), Buffone et al. (2004), Sefiane and Ward (2007) among others. On the other hand, experimental investigations of Buffone et al. (2005) demonstrated the thermocapillary convection self-induced from vertical interface where an interesting results is that the convection is driven by the surface-tension gradient generated by the self-induced temperature difference. A similar situation is proposed in the present work, where we have a vertical interface and thermocapillary convection driven by evaporation is self-induced, as opposed to thermocapillary convection under imposed temperature gradients. Despite mentioned above, film boiling has received much less attention than thermocapillary-nucleate boiling and not works theoretical and/or experimental have been reported as far as the author know.

Then, as mentioned above, yet the thermocapillary-film-type phenomenon appears susceptible to attack from the theoretical viewpoint.

Here, an attempt is made to relate the peculiar behavior of the heat transfer in vertical geometries with the thermocapillary phenomenon, where nowadays physical models is still missing.

1.1. Pool film boiling

At or near boiling crisis a film boiling occurs when the heat surface is blanketed with vapor film and the heat transfer coefficient rapidly decreases, the film hinders heat transfer, and resulting heat flux is usually small compared with the values observed during nucleate boiling . In most industrial process for vaporizing liquids, film boiling is avoided. However, when large temperature differences are encountered, such as when an ordinary liquid contacts a very hot solid or when vapor is used to heat liquids with low boiling points, film boiling may occur.

2. Theoretical background

On the presence of a thermal gradient, there are two driving forces. In one hand

- Thermal expansion.

On the other hand

- Temperature surface-tension coefficient.

The former effect-thermal convection, scaled by the dimensional Grashof number is proportional to the fourth power layer thickness; hence, for small thickness the thermocapillary convection dominates. In the present paper, where the film boiling can be considered as very small, the thickness of a film boiling in first approximation can be calculated as

$$\zeta \approx \frac{\kappa_v}{h} \tag{1}$$

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Nomenclature

b	*-in* growth coefficient
g	gravity constant
Gr	Grashof number: $Gr = \alpha g l^3 \Delta / \nu^2$
h	heat transfer coefficient
H	liquid depth
i	co-ordinate parallel to the boundary layer
k	wave number
l	characteristic length
m	wave number
n	wave frequency
p	pressure
R	Radius of curvature of the interface
t	time
T	temperature
u	velocity
x	co-ordinate parallel to the wall
z	co-ordinate normal to the wall $\Delta T_{sat} = T_W - T_{sat}$
$\nabla_i T$	thermal gradient in the parallel direction to the interface vapor–liquid.

Greeks

α	thermal dilatation coefficient $\alpha = (1/\rho)\partial\rho/\partial T$
β	expansion term : $\beta = 1 - \frac{\rho_v}{\rho_l}$
κ	thermal conductivity
η	dynamical viscosity
λ	wave length
ν	kinetic viscosity
η	distance perpendicular to liquid–vapor interface
ω	angular frequency
ρ	density
σ	surface tension
σ'	temperature surface-tension coefficient $\sigma' = \frac{\partial\sigma}{\partial T}$
ζ	vapor film thickness
Δ	temperature differentia

Subscripts

d	the "most dangerous" wave length defined by Eq. (27)
f	film
v	vapor
l	liquid
vl	vapor–liquid interface
s	the "smallest" wave defined by Eq. (26)
sat	saturation
sub	subcooled
w	wall
0	initial-reference value

where ζ and κ_v are the vapor film thickness and thermal conductivity, respectively and the h is the heat transfer coefficient. Here, we have in mind thin films above $h \approx 300\,\text{W/m}^2\,\text{K}$, i.e., vapor film thickness of the order of $\zeta \leqslant 10^{-5}$ m, in this case thermocapillary convection can be largely dominant as it will be demonstrated.

3. Basic assumptions

We have a heat surface which is blanketed with a vapor film with constant density ρ_v; surface tension σ_f; and velocity u_v, the film is separated of the rest of the saturated liquid (upper region) by the boundary layer, which is at saturation temperature and the upper region (saturated liquid) with constant density ρ_l, surface tension σ_l, and velocity u_l.

The actual shape of the liquid–vapor interface, and the co-ordinates in the model used in the analysis are shown in Fig. 1. Additionally, radiative heat transfer is neglected, because it is negligible comparing to the conductive heat transfer in temperature less to 1200 K. Furthermore it is supposed that the velocity field of the film is parallel to the axis "*x*".

For the momentum equation

$$\eta \frac{\partial^2 u}{\partial z^2} = \frac{\partial p}{\partial x} \tag{2}$$

where η is the dynamic viscosity, u the velocity and p the pressure. If the Eq. (2) is discomposed for the vapor and liquid, in this case we have

$$\begin{aligned} \eta_v \frac{\partial^2 u_v}{\partial z^2} &= \frac{\partial p_v}{\partial x} \\ \eta_l \frac{\partial^2 u_l}{\partial z^2} &= \frac{\partial p_l}{\partial x} \end{aligned} \tag{3}$$

with the boundary conditions (in the heater surface):

$$u_v = 0; \quad z = 0 \tag{4}$$

and in the same manner if our system is bounded by walls both at $z = 0$ and $z = H$, "*the rigid case*", we have

$$u_l = 0; \quad z = H \tag{5}$$

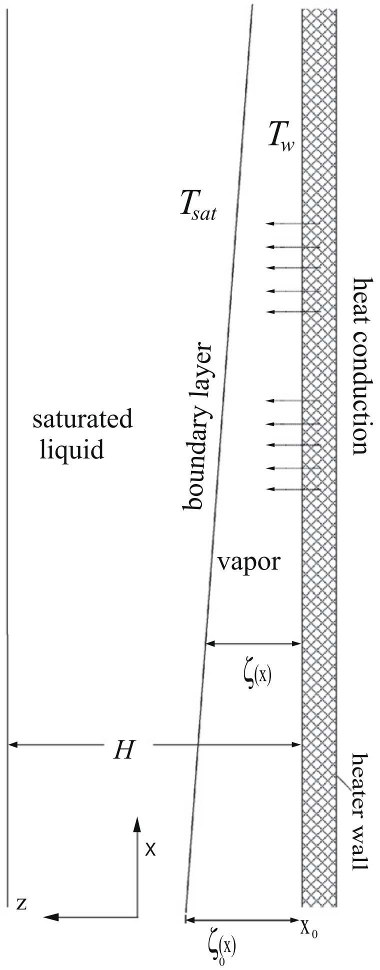

Fig. 1. Physical model and co-ordinates system.

In the interface vapor–liquid $z = \zeta$, must fulfill the boundary condition (Landau and Lifshitz, 1991),

$$\eta_v \frac{\partial u_v}{\partial z}\bigg|_{\zeta} = \nabla_x T \sigma'_f$$
$$\eta_l \frac{\partial u_l}{\partial z}\bigg|_{\zeta} = \nabla_x T \sigma'_l \tag{6}$$

where $\nabla_x T$ is the temperature gradient along the interface, and of course, we have assumed implicitly that the interface vapor liquid is practically parallel along $0x$ axis, and σ' is referred to the surface-tension temperature coefficient $\sigma' = \frac{d\sigma}{dT}$.

For the stationary flow, the mass flow across a straight section of the layer must be zero i.e.,

$$\int_0^{\zeta} u_v = 0; \qquad \int_{\zeta}^{H} u_l = 0 \tag{7}$$

where, of course, the vapor mass flow in the microregion of evaporation is confined in a very small width in the interface and parallel to the interface, hence dynamical and kinematics effects are not significants. with the conditions (4)–(7) applied to Eq. (3) we have finally

$$\frac{\partial p_v}{\partial x} = \frac{3}{2}\frac{\sigma'_f}{\zeta}\nabla_x T; \qquad \frac{\partial p_l}{\partial x} = \frac{3}{2}\frac{\sigma'_l}{(H-\zeta)}\nabla_x T \tag{8}$$

if we define a total surface-tension temperature coefficient as:

$$\sigma' = \sigma'_f + \sigma'_l \tag{9}$$

and taking into account that in the interface $z = \zeta$ the velocity is given by

$$u_v = \frac{\zeta\sigma'_f}{4\eta_v}\nabla_x T = u_l = \frac{(H-\zeta)\sigma'_l}{4\eta_l}\nabla_x T \tag{10}$$

we obtain the following relation between the surface-tension temperature coefficients

$$\sigma'_l = \frac{\sigma'}{1 + \frac{\eta_v}{\eta_l}\frac{H-\zeta}{\zeta}} \tag{11}$$

$$\sigma'_f = \frac{\sigma'}{1 + \frac{\eta_l}{\eta_v}\frac{\zeta}{H-\zeta}} \tag{12}$$

A simplified assumption can be introduced (11) and (12) when the vapor thickness is very small (our case), assuming ($H \gg \zeta$), in this case and taking into account Eq. (8) the pressure gradient $\frac{\partial p_v}{\partial x}$, is much larger than $\frac{\partial p_l}{\partial x}$ and taking into account Eq. (8) the differentia reduces to

$$\frac{\partial p_v}{\partial x} - \frac{\partial p_l}{\partial x} = \frac{3}{2}\frac{\sigma'}{\zeta}\nabla_x T \tag{13}$$

Now, take two neighboring points A and B on the interface, in the (x, z) plane $x_B - x_A = \delta_x$; we may write

$$p_B = p_A + \frac{\partial p_v}{\partial x}\delta_x + \Delta P_\sigma = p_A + \frac{\partial p_l}{\partial x}\delta_x \tag{14}$$

where the term P_σ takes into account the pressure differentia due to extremely small curvature of the interface and the surface tension, a simple derivation is given in Appendices A and B.

$$\Delta P_\sigma = \frac{2}{3}(\rho_l - \rho_v) g \frac{d\zeta}{dx}\delta x \tag{15}$$

so, inserting Eq. (15) in Eq. (14) the interface slope yields the following result.

$$\frac{d\zeta}{dx} = -\frac{9}{4}\frac{\sigma'\nabla_x T}{g\zeta(\rho_l - \rho_v)} \tag{16}$$

assuming that the origin of x is chosen to be the origin of virtual 3origin of the vapor layer $\zeta = \zeta_0$, integrating the expression (16) between $(0, \zeta_0)$ and (x, ζ), we have

$$\frac{\zeta}{\zeta_0} = \left[1 - \frac{9\sigma'\nabla_x T}{2\zeta_0^2 g(\rho_l - \rho_v)}x\right]^{\frac{1}{2}} \tag{17}$$

In vertical film boiling the thermal gradient in the boundary layer (saturation temperature gradient) is due to the differential in hydrostatic pressure along $0x$ axis. Then

$$\nabla_x T = \nabla_p T_{\text{sat}} \cdot \frac{\partial p}{\partial x} = -\nabla_p T_{\text{sat}} \cdot \rho_l g \tag{18}$$

inserting Eq. (18) in Eq. (17) and defining a expansion term as $\beta = 1 - \frac{\rho_v}{\rho_l}$ and taking into account that the temperature surface-tension coefficient σ', is practically always negative, due to the entropy term in the surface free energy, Eq. (17) can be expressed in a more compact form

$$\frac{\zeta}{\zeta_0} = \left[1 - \frac{9|\sigma'|}{2\zeta_0^2\beta}\nabla_p T_{\text{sat}}\, x\right]^{\frac{1}{2}} \tag{19}$$

A heat transfer coefficient may be defined by applying the following equation

$$h = \frac{\kappa_v}{\zeta} \tag{20}$$

in first approximation it is plausible to assume superposition of the thermocapillary phenomena in the average thickness film ζ_0, with this assumption the heat transfer coefficient corrected by thermocapillary gives,

$$h = h_0 \cdot \left[1 - \frac{9|\sigma'|}{2\zeta_0^2\beta}\nabla_p T_{sat} x\right]^{-\frac{1}{2}} \tag{21}$$

where $h_0 = \frac{\kappa_v}{\zeta_0}$.

4. Experimental data

It is interesting to compare the above result with that experimental data of Shiotsu and Hama (2000) for R113 coolant. Fig. 2 shows the comparison between data of h for R113 under saturated condition at the system pressure of 490 kPa versus the surface superheat, ΔT_{sat} and Eq. (21) taking into account thermocapillary

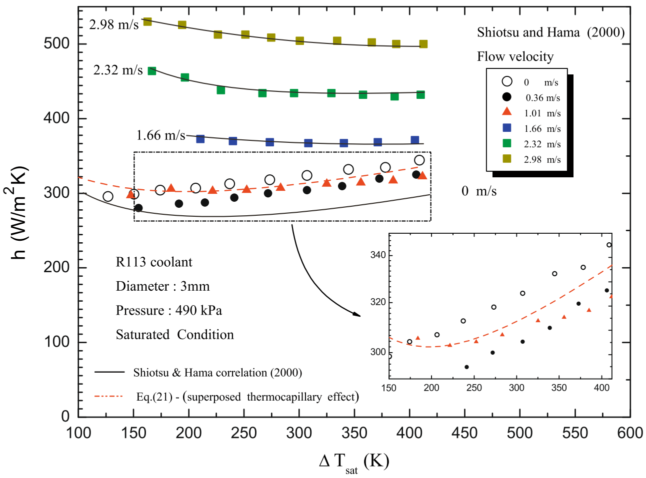

Fig. 2. Film boiling heat transfer coefficient h versus wall superheat for R113 under saturated condition at 490 kPa by Shiotsu and Hama (2000) and the thermocapillary correction Eq. (21).

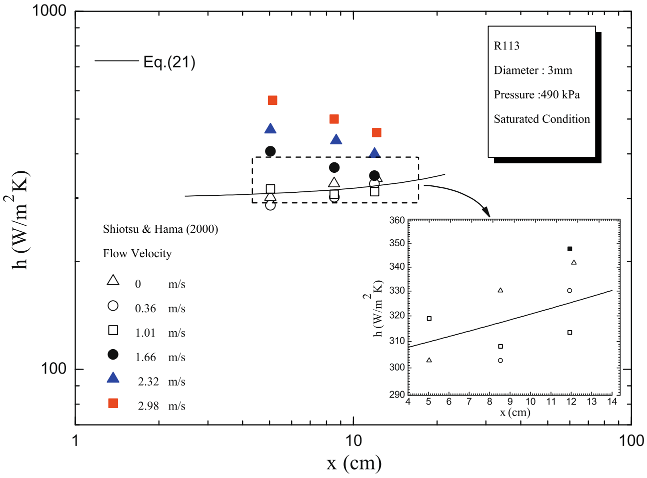

Fig. 3. Axial variation of film boiling heat transfer coefficients in R113 for the wall superheated of 300 K at pressure of 490 kPa versus vertical position *x*.

effects. The heat transfer coefficients for R113: the values at a certain surface superheat are almost independent of the velocities $u_l < 1$ m/s and increase with the increase in flow velocity from the value. Fig. 3 shows the comparison between data of *h* axial variation of film boiling heat transfer coefficients in R113 coolant for the wall superheat of 300 K at pressure of 490 kPa versus vertical position *x*, and different velocity u_l and Eq. (21) taking into account thermocapillary effects. Referring to Fig. 3 as mentioned above, the heat transfer coefficients for the first range $u_l < 1$ m/s of flow velocities are almost independent of *x*. The apparent paradoxical behavior in the experimental results of *x*-independent heat transfer coefficients were already reported in pool film boiling on vertical surfaces by some workers. Bui and Dhir (1985), Liaw and Dhir (1986) in a pool film boiling heat transfer coefficients on the 63-mm wide and 103-mm high isothermal vertical plate in saturated water and R113 at atmospheric pressure. Their heat transfer coefficients at the distance longer than 26 mm were almost in agreement with each other, although that at $x = 8$ mm was slightly higher than the others. Sakurai et al. (1990), Sakurai and Shiotsu (1992) analyzed the conventional data of pool film boiling heat transfer coefficients on vertical plates and rods and reported that they were almost independent of the vertical position for the non-dimensional length from the leading edge of the vertical surface $x' = 2\pi\frac{x}{\lambda_s}x \geqslant 7$ the authors proposed that an unstable wave caused by Helmholtz instability would become dominant for the length larger than the wave length of the instability; the disturbances would increase the heat transfer coefficients and make them almost independent of x'.

Hence, the results of the present paper are an alternative and new theoretical viewpoint for the enhancement in the heat transfer coefficient in low velocity and in absence of Helmholtz instabilities (stationary film boiling). The good agreement with experimental data provides added confidence in the validity of Eq. (21). However, it is evident that the thermocapillary phenomena can contribute to the apparition and growth of Helmholtz instabilities and both models: Helmholtz instability, and thermocapillary convection could be complementary.

5. Summary of results and conclusions

(a) It is qualitatively reasonable to suppose that if the vapor thickness is very small Eq. (17), thermocapillary convection together with bulk thermal convection can enhance the heat transfer in saturated conditions.

(b) A thermocapillary factor Eq. (21) is derived, taking into account thermocapillary convection in a flat film boiling and showing the strong dependence with the vapor film thickness. The result is an alternative and new theoretical viewpoint for the enhancement in the heat transfer coefficient observed in the experimental data in low velocity field.

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Appendix A. Derivation of the pressure difference due to curvature of the interface

The pressure difference due to the curvature of the interface is

$$P_\sigma = \frac{2\sigma}{R} \tag{22}$$

where *R* is the radius of curvature of the interface. In order to determine this pressure due to surface tension, the radius of curvature of the interface is first deduced. In a two-dimensional approximation, suppose that the wave surface is described by the equation (Milne and Thompson, 1996):

$$z = \zeta \sin \kappa x \tag{23}$$

where $\kappa = 2\pi/\lambda_m$. The following expression is therefore obtained for the maximum curvature of the surface

$$\frac{1}{R} = \left|\frac{\partial^2 z}{\partial x^2}\right| = \frac{4\pi^2\zeta_0}{\lambda_m^2} \tag{24}$$

where λ_m is the wavelength on the surface a neutral oscillation. During film boiling under natural convection conditions the vapor film is small, and hence as shown in Kutateladze and Nakoryakov (1984), capillary-gravitational waves arise on the surface which can be described by the following dispersion relationship:

$$\omega = \left[\frac{gk(\rho_l - \rho_v) + \sigma k^3}{\rho_l + \rho_v}\right]^{\frac{1}{2}} \tag{25}$$

in Eq. (25) we obtain the limits for the instability development i.e., the smallest which can grow in amplitude λ_s (i.e., b=0) given by

$$\lambda_s = 2\pi\sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}} \tag{26}$$

and the "*most dangerous*" wave, referring to the wave which increases its amplitude most rapidly given by the following:

$$\lambda_d = 2\pi\sqrt{\frac{3\sigma}{g(\rho_l - \rho_v)}} \tag{27}$$

It can be assumed that Eq. (26) remains valid for the conditions of film boiling being considered up to the onset of static instability. Then by making use of Eqs. (24) and (26) in Eq. (22), an expression is obtained for the surface-tension pressure:

$$P_\sigma = \frac{2}{3}(\rho_l - \rho_v)g\zeta \tag{28}$$

Appendix B. Flatness and stability to film boiling

In this appendix, the condition of flatness and stability in front of Helmholtz instabilities as a natural consequence of a vapor film thickness very small will be demonstrated, i.e., it is not additional condition and hence the present model is self-consistent.

The form of the interface disturbances introduced into the first-order perturbation analysis is given by the following equation.

$$\eta = \eta_0 e^{bt} \cos mx \tag{29}$$

the meaning of the various terms in the above equation are defined in the Nomenclature. The general solution kinematic equation yields the following relation between wave speed, gravity, surface tension for the case of a liquid over a vapor in low velocity valid for a film boiling when the vapor thickness is very small (Millne-Thompson, 1996) (the reader can find a simple derivation in the well-known work of Berenson (1961) for Taylor–Helmholtz instability in film boiling).

$$b = \left[\frac{g(\rho_l - \rho_v)m}{\rho_l + \frac{\rho_v}{m\zeta}} - \frac{g_0 \sigma m^3}{\rho_l + \frac{\rho_v}{m\zeta}}\right]^{\frac{1}{2}} \tag{30}$$

Thus, it is easy to see in the above equation, that for a vapor thickness very small, the instability growth parameter b is strongly reduced, and the probability of an unstable interface, that is, a disturbance growing with time is very restricted. In the same manner, long-wavelength ($m \to 0$) decrease the growth parameter.

Additionally, Budov et al. (1983) examined the instability of film boiling of a moving liquid, and they found that the wavelength increase with further decrease of the liquid velocity, in other words in the viscous region (low velocity) we find a predisposition to long-wavelengths.

As mentioned above, then the following assumption is correct: "*in the viscous region (low velocity) and a vapor film thickness very small where additionally predominate long-wavelengths, the Helmholtz instability is strongly restricted i.e., we have high stability in front of Helmholtz instabilities*".

Now the demonstration of "flatness" for film boiling is straightforward, taking into account the above deduction in the curvature Eq. (24) it is seen that for a very small film boiling and further long-wavelengths the curvature $\frac{1}{R}$ is very small i.e., to obtain some idea of the dimensions of the curvature, for a film boiling above 10^{-3} mm and typical wavelengths of 18 mm we obtain for the amplitude of oscillation Eq. (28) $a \approx \frac{2}{3}\zeta = 6 \times 10^{-4}$ mm and for the radius of curvature Eq. (24) $R \approx 820$ mm in other words, the condition of flatness is strongly justified (*Q.E.D*).

Then, according to the present theoretical model, thermocapillary is possible with very small perturbations, the above theoretical conclusion agree with several authors (Bennacer et al., 2004; Sefiane and Ward, 2007) among others, where in computational simulations showed that a minimum irregularity in evaporative flux along the liquid–vapor interface is sufficient for the onset of thermocapillary convection.

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